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PROBLEM OF THE SYNTHESIS OF A COMPOSITE MATERIAL OF UNIDIMENSIONAL STRUCTURE WITH ASSIGNED CHARACTERISTICS

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A significant amount of attention is now being given to the development of composite materials with assigned properties. Here we present a solution of this problem in regard to the thermophysical and stiffness characteristics of composites with a unidimensional structure (with the condition that the components have the same Poisson's ratios).

By a composite material with a unidimensional structure we mean an inhomogeneous material with thermophysical and mechanical characteristics which are a function of a single space variable, such as x_1. Composites constitute a special case of such materials. The characteristics of a composite composed of a large number of small components are rapidly oscillating functions with a characteristic magnitude of oscillation $\varepsilon \ll 1$ (in the case of laminated composites, ε is the characteristic thickness of the layers). As was shown in [1-5], at $\varepsilon \to 0$ an inhomogeneous composite with a periodic structure can be regarded as a homogeneous material with so-called averaged [1-5] thermophysical and mechanical characteristics which at $\varepsilon \ll 1$ are close to the thermomechanical behavior of the original material [1-6]. The averaged characteristics, describing the material from the macroscopic viewpoint, are determined by its its local (microscopic) characteristics. The question of determining averaged characteristics of composites from their local characteristics has been fully resolved by now [1-7]. Here we examine the inverse problem: through which averaged characteristics and in what manner can we impart a unidimensional structure to composites by controlling their local characteristics? The solution is obtained on the basis of the methods used in [8, 9] in regard to thermophysical and stiffness characteristics.

Let the composite material we are studying be locally isotropic and inhomogeneous, with a periodic structure. The characteristic size of the period $\varepsilon \ll 1$. We apply the following restriction to the types of composites for which our findings are valid: the materials used in the composite must have the same (or similar) Poisson's ratios. This condition is met, for example, by a composite based on metals ($\nu \approx 1/3$) or polymers ($\nu \approx 0.4$). The material characteristics of the composites being examined: $c(x_1/\varepsilon)$, $a(x_1/\varepsilon)$ are the local heat capacity and thermal conductivity; $E(x_1/\varepsilon)$, $A(x_1/\varepsilon)$ are the local Young's modulus and coefficient of linear expansion [the period of the functions $c(t)$, $a(t)$ $E(t)$, $A(t)$ is equal to unity]. At $\varepsilon \to 0$, the solutions of the heat-conduction and strain problems for the composite approach [in the norm of the space $L_2(Q)$] the solutions of the same problem for a homogeneous anisotropic material with averaged characteristics:

heat capacity [2, 7]

$$\hat{c} = \langle c \rangle, \tag{1}$$

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where $\langle f\rangle = \frac{1}{\varepsilon}\int\limits_0^{\varepsilon} f(x_1/\varepsilon)\,dx_1 = \int\limits_0^1 f(t)\,dt$ is the mean for the period;

the thermal conductivity tensor [1, 2]

$$\hat{a}_{11} = 1/\langle 1/a\rangle,\ \hat{a}_{22} = \hat{a}_{33} = \langle a\rangle \tag{2}$$

(the tensor components not explicitly indicated are equal to zero to within the known symmetries);

the compliance tensor [7]

$$\begin{gathered}
H_{1111} = \frac{(1+\nu)(1-2\nu)}{1-\nu}\left\langle\frac{1}{E}\right\rangle + \frac{2\nu^2}{1-\nu}\frac{1}{\langle E\rangle},\\
H_{2222} = H_{3333} = \frac{1}{\langle E\rangle},\quad H_{1122} = H_{1133} = H_{2233} = -\frac{\nu}{\langle E\rangle},\\
H_{1313} = H_{1212} = \frac{1+\nu}{2}\left\langle\frac{1}{E}\right\rangle;\quad H_{2323} = \frac{1+\nu}{2}\frac{1}{\langle E\rangle};
\end{gathered} \tag{3}$$

the tensor of the coefficients of linear expansion [6]

$$\begin{gathered}
A_{11} = \frac{1+\nu}{1-\nu}\langle A\rangle - \frac{2\nu}{1-\nu}\frac{\langle EA\rangle}{\langle E\rangle},\\
A_{22} = A_{33} = \frac{\langle EA\rangle}{\langle E\rangle}.
\end{gathered} \tag{4}$$

Equations (1)-(4) establish the relationship between the averaged (macroscopic) and the local (microscopic) characteristics of composites with a unidimensional structure. In general, the distribution of local characteristics in composites may be quite different in character from one case to another: continuous, piecewise-continuous, piecewise-constant (the latter occurs in laminated composites), etc. To cover such cases, we introduce the following set of functions: $W = \{f(t) \in L_\infty([0,\ 1]):\ 1/f(t) \in L_\infty([0,\ 1])$, and for any f(t) we find a number $\xi(f) > 0$ such that $f(t) \geq \xi(f)$ for nearly all $t \in [0,\ 1]\}$.

The set W contains the above types of functions, which nearly exhausts the set of possible distributions of local characteristics of composites. Thus, we will henceforth assume that the set of functions (c, a, E, A)(t) – the local characteristics of the composite – belongs to the set W^4. Accordingly, Eqs. (1)-(4) give the mapping $I: (c,\ a,\ E,\ A) \in W^4 \to R^{16}$ from the set of distribution of local characteristics onto the set of values of the averaged characteristics. We will use $I(W^4)$ to designate the image of the set W^4 in the mapping I, $I(W^4) = \{\mathbf{x} \in R^{16}$: there exists a set of functions $(c,\ a,\ E,\ A)(t) \in W^4$ such that $I(c,\ a,\ E,\ A) = \mathbf{x}\}$.

The problem of designing a composite with a unidimensional structure and assigned averaged characteristics is formulated as follows: we want to have a material possessing a prescribed set of thermophysical and stiffness characteristics: $c^0,\ a^0_{ij},\ H^0_{ijkl},\ A^0_{ij}$. We need to do the following: 1) determine whether or not a material with such characteristics can be created in the class of composites with a unidimensional structure; 2) if the first question is answered affirmatively, we need to determine the method by which such a material can be created. It is not hard to see that a material with a prescribed set of averaged characteristics can be created within the class of composites with a unidimensional structure only when $(c^0,\ a^0_{ij},\ H^0_{ijkl},\ A^0_{ij}) \in I(W^4)$. This means that to solve the first problem, it is sufficient to give a description of the set $J(W^4)$. The solution of the second problem reduces to the following: for each element $x \in I(W^4)$, we need to indicate the method of construction of the set of functions $(c,\ a,\ E,\ A)(t) \in W^4$ – local characteristics of the composite – such that $I(c,\ a,\ E,\ A) = x$ (i.e., we need to solve the synthesis problem in [8, 9] for the mapping I).

Let us proceed to the solution of the stated problem. We note that Eqs. (1), (2) and (3), (4) are independent and that the averaged characteristics which they give are in one-to-one correspondence with functionals of the form

$$\int\limits_0^1 u_1(t)\,dt,\quad \int\limits_0^1 \frac{dt}{u_1(t)},\quad \int\limits_0^1 u_2(t)\,dt,\quad \int\limits_0^1 u_1(t)\,u_2(t)\,dt \tag{5}$$

[for (3), (4) $u_1(t) = E(t)$, $u_2(t) = A(t)$; Eqs. (1) and (2) include functionals of the first and second types]. By virtue of the foregoing, it will be sufficient to solve problems 1 and 2 for the mapping J of the set W^2 onto R^4 given by Eqs. (5). Let us obtain the solution of this problem. Here, the functions $(u_1(t),\ u_2(t)) \in W^2$ will be regarded as the control, assigned on a segment with the ends $t_0 = 0$, $t_1 = 1$, and we will use the results in [8, 9]. During the course of the solution, we require the auxiliary set $W_\xi = \{f(t) \in W : f(t) \geqslant \xi$ for nearly all $t \in [0;\ 1]\}$, $\xi > 0$ – is a parameter. It is obvious that $W_\xi \subset W$ for any $\xi > 0$.

It is easily seen that the image of the set W_ξ^2 in the mapping $J_3 : W^2 \to R^3$, given by the first three functionals in (5), is $J_3(W_\xi^2) = \{(x,\ y,\ z) \in R^3 : x \geqslant \xi,\ 1/x \leqslant y \leqslant 1/\xi,\ z \geqslant \xi\}$. The image of the set W^2 with the same mapping is obtained by replacing ξ by 0 and $1/\xi$ by ∞.

To calculate the image of the set W_ξ^2 in the mapping J (5), first we examine the problem of minimization of the fourth functional in (5) with given values of the first three functionals:

$$\int_0^1 u_1(t)\, u_2(t)\, dt \to \inf(\xi); \tag{6}$$

$$\int_0^1 u_1(t)\, dt = a, \quad \int_0^1 \frac{dt}{u_1(t)} = b, \quad \int_0^1 u_2(t)\, dt = c, \tag{7}$$

where $a > \xi;\ 1/a < b < 1/\xi;\ c > \xi;$

$$(u_1(t),\ u_2(t)) \in W_\xi^2. \tag{8}$$

Problem (6)-(8) is a Lyapunov problem [9] with restrictions on the control in the form of inequalities [condition (8)]. In accordance with the maximum principle [9, p. 354], for its solution $\mathbf{U}(t) = (U_1(t),\ U_2(t))$, if a solution exists, the Lagrangian $K(\mathbf{u}(t),\ \boldsymbol{\lambda}) = \lambda_0 u_1(t) u_2(t) + \lambda_1 u_1(t) + \lambda_2/u_1(t) + \lambda_3 u_2(t)$ satisfies the following equality with the appropriate choice of Lagrangian multipliers $\lambda_0 \geqslant 0,\ \lambda_i$ (i = 1, 2, 3)

$$K(\mathbf{U}(t),\ \boldsymbol{\lambda}) = \min_{\mathbf{u} \in W_\xi^2} K(\mathbf{u}(t),\ \boldsymbol{\lambda}) \tag{9}$$

for nearly all $t \in [0,\ 1]$. We take $\lambda_0 = 1,\ \lambda_1 = \lambda_3 = -\xi,\ \lambda_2 = 0$. Then the equality in (9) is achieved on functions taking values in the set $V_\xi = \{(x,\ y) \in R^2 :\ x = \xi,\ y \geqslant \xi$ (any) or $x \geqslant \xi$ (any), $y = \xi\}$. In paticular, the equality is satisfied on the function $\mathbf{U}(t) = (U_1(t),\ U_2(t))$ of the form

$$U_1(t) = \begin{cases} X & \text{at} \quad t \in [0,\ H], \\ \xi & \text{at} \quad t \in (H,\ 1], \end{cases} \quad U_2(t) = \begin{cases} \xi & \text{at} \quad t \in [0,\ H], \\ Y & \text{at} \quad t \in (H,\ 1], \end{cases} \tag{10}$$

where $0 \leqslant H \leqslant 1;\ X,\ Y \geqslant \xi$. For the function (10), Eqs. (7) can be satisfied by the selection of X, Y, and H. In fact, for $U_1(t)$, $U_2(t)$, given by (10), Eqs. (7) are

$$\begin{gathered} XH + \xi(1 - H) = a > \xi, \\ H/X + (1 - H)/\xi = b \in (1/a,\ 1/\xi), \\ \xi H + Y(1 - H) = c > \xi. \end{gathered} \tag{11}$$

From the first equation $H = (a - \xi)/(X - \xi) \in [0,\ 1]$. Having inserted this expression into the left side of the second equality in (11), we see that it is $(X - \xi)/X(X - \xi) + (X - a)/\xi(X - \xi)$. The values of the resulting function at $X \in (a,\ \infty)$ cover the interval $(1/a,\ 1/\xi)$, so that the first two equations in (11) can be solved. The third equation can always be satisfied by appropriate selection of $Y \geqslant \xi$. Thus, for the function $\mathbf{U}(t) = (U_1(t),\ U_2(t))$, with the selection of X, Y, and H as solutions of system (11), the maximum principle (9) and conditions (7), (8) are satisfied. Also, $\lambda_0 > 0$, and on the set W_ξ ($\xi > 0$) the integrands in (6), (7) are continuous with respect to u_1, u_2. Then Part 2 of the theorem in [9, p. 354] can be used. As a result of this theorem, $\mathbf{U}(t)$ (10) is the solution of problem (6)-(8). We find from (9) that the minimum value of the integral (6) is $\inf(\xi) = -\xi^2 + a\xi + b\xi$.

Now we will show that with the satisfaction of conditions (7), (8), the integral (6) can take any value greater than $\inf(\xi)$. As a result, the image of the set W_ξ^2 in the mapping J (5) contains the set $Z_\xi = \{(x,\ y,\ z,\ t) \in R^4 :\ x > \xi,\ 1/x < y < 1/\xi,\ z > \xi,\ t > -\xi^2 + x\xi + y\xi\}$. To achieve this, we introduce a special acicular variation $\mathbf{V}(t)$ of the function $\mathbf{u}(t) \in W_\xi^2$, determined by the formula

$$V_i(t) = \begin{cases} u_i(t(1 + \delta)) & \text{at} \quad t \in [0, 1/(1 + \delta)], \\ K_i \delta^{-1/2} & \text{at} \quad t \in (1/(1 + \delta),\ 1], \end{cases} \quad i = 1, 2. \tag{12}$$

Here, $\infty > C \geqslant K_1, K_2 \geqslant 0, \delta \geqslant 0$ are parameters satisfying the condition $K_i\delta^{-1/2} \geqslant \xi$ (i = 1, 2), by virtue of which $\mathbf{V}(t) \in W_\xi^2$. If we use $(a, b, c) = J_3(\mathbf{u}(t))$ to designate values of the first three functionals on the function $\mathbf{u}(t) \in W_\xi^2$, then

$$\int_0^1 V_1(t)\,dt = \frac{a}{1+\delta} + K_1\delta^{\frac{1}{2}}, \quad \int_0^1 \frac{dt}{V_1(t)} = \frac{b}{1+\delta} + \frac{\delta^{\frac{1}{2}}}{K_1},$$
$$\int_0^1 V_2(t)\,dt = \frac{c}{1+\delta} + K_2\delta^{\frac{1}{2}}, \quad \int_0^1 V_1(t)\,V_2(t)\,dt = \frac{-\xi^2 + a\xi + b\xi}{1+\delta} + K_1K_2. \tag{13}$$

The right sides of (13) give the function $\mathbf{r}(a, b, c, d, K_1, K_2, \delta)$ with the variable δ, where $a, b, c, d = -\xi^2 + a\xi + b\xi + K_1K_2$ and K_1, K_2 are parameters. It suffices to show that for any $(x, y, z, t) \in Z_\xi$ the equation $\mathbf{r}(a, b, c, d, K_1, K_2, \delta) = (x, y, z, t)$ is solvable at $\delta = \delta_0 > 0$ through suitable selection of $\mathbf{u}(t) \in W_\xi^2$ and the quantities δ_0, K_1, K_2. It is evident from (13) that the equation $\mathbf{r}(a, b, c, d, K_1, K_2, \delta) = (x, y, z, t)$ is solvable relative to the parameters a, b, c, d in the form $(a, b, c, d) = \mathbf{r}^{-1}(x, y, z, t, K_1, K_2, \delta)$, where the function $\mathbf{r}^{-1}(x, y, z, t, K_1, K_2, \delta) \to (x, y, z, t)$ at $\delta \to 0$ and is continuous with respect to δ. By virtue of $(x, y, z, t) \in Z_\xi$ and the openness of the set Z_ξ, beginning with a certain $\delta = \delta_0 > 0$, the point (a, b, c) belongs to the interior of the set $J_3(W_\xi^2)$. Thus, for these (a, b, c), problem (6)-(8) is solvable. We take the solution of problem (6)-(8) as $\mathbf{u}(t)$ in (12), while we choose K_1 and K_2 from the condition $K_1\delta_0^{-1/2} \geqslant \xi$, $K_2\delta_0^{-1/2} \geqslant \xi$, $C \geqslant K_1$, $K_2 \geqslant 0$, $K_1K_2 = d - (-\xi^2 + a\xi + b\xi)$ [since $(a, b, c, d) \in Z_\xi$, beginning with a certain $\delta_0 > 0$, then at $\delta < \delta_0$ the quantity $d - (-\xi^2 + a\xi + b\xi) > 0$, and the above system is solvable]. Consequently, for variation (12) of the function U(t) with the given choice $\delta_0 > 0$ and K_1, K_2, the equality $J(\mathbf{V}(t)) = (x, y, z, t)$ is satisfied. Let us sum up the results obtained.

Proposition 1. a) The image of the set W^2 in the mapping J given by (5) coincides with the set $Z = \{(x, y, z, t) \in R^4\colon x > 0,\ y > 1/x,\ z > 0,\ t > 0\}$ to within the interior points of the set; b) any point of the set Z can be obtained as the value of the mapping J (5) on a piecewise-constant function taking no more than three different values.

Case "a" of Proposition 1 is easily obtained by taking the limit at $\xi \to 0$ from the previous results. As regards "b," we note that one method of constructing the function in it was presented above [this is the variation (12) of the function $\mathbf{U}(t)$, determined from the solution of problem (6)-(8) with the corresponding selection of the values $\delta_0 > 0$, K_1, K_2].

Let us proceed to the mechanical interpretation of the results obtained. Proposition 1 makes it possible to describe the set of all possible averaged characteristics of composites with a unidimensional structure (problem 1) and to establish a method for making (synthesizing) composites with any of the possible characteristics (problem 2).

Proposition 2. a) Composites with a unidimensional periodic structure made of isotropic components can have the following averaged characteristics and cannot have other characteristics (to within the boundaries of the enumerated sets):

heat capacity

$$\hat{c} = X \ (X > 0); \tag{14}$$

the thermal conductivity tensor

$$\hat{a}_{11} = Y,\ \hat{a}_{22} = \hat{a}_{33} = Z \ (Z > 0,\ Y > 1/Z); \tag{15}$$

the Young's moduli E_i, Poisson's ratios ν_{ij}, and shear moduli G_{ij}

$$E_1 = \frac{(1-\nu)\,x}{(1+\nu)(1-2\nu)\,xy + 2\nu^2}, \quad E_2 = E_3 = x,$$
$$\nu_{12} = \nu_{13} = \frac{\nu(1-\nu)}{(1+\nu)(1-2\nu)\,xy + 2\nu^2}, \quad \nu_{23} = \nu, \tag{16}$$
$$G_{12} = G_{13} = \frac{2}{(1+\nu)\,y}, \quad G_{23} = \frac{2x}{1+\nu};$$

the tensor of the coefficients of linear expansion

$$A_{11} = \frac{1+\nu}{1-\nu}z - \frac{2\nu}{1-\nu}\frac{t}{x}, \quad A_{22} = A_{33} = \frac{t}{x}$$
$$(x > 0,\ y > 1/x,\ z > 0,\ t > 0). \tag{17}$$

The variables X, Y, Z, x, y, z, t in (14)-(17) take independent values within the ranges indicated for them (the tensor components not explicitly indicated are equal to zero to within the known symmetries); b) any of the possible averaged characteristics indicated in (14)-(17) can be obtained as averaged characteristics of a laminated composite formed of no more than three different materials. Proposition 2 is a direct consequence of Proposition 1 and Eqs. (1)-(4).

Note 1. By a laminated composite we mean a composite with a unidimensional structure and having local characteristics which are piecewise-constant functions. This class of composites includes composites made by welding, gluing, layer-by-layer deposition, etc. for layers of homogeneous materials. It follows from Proposition 2 (case "b") that any possible averaged characteristics of composites with a unidimensional structure (particularly composites with a continuous distribution of the local characteristics) can be obtained in the class of laminated composites — which are the easiest composites to make.

Note 2. It is also possible to use more than three materials to make the composites, of course. However, the use of more than three materials will not expand the set of values of the averaged characteristics that can be achieved.

Proceeding on the basis of Proposition 2, we propose the following procedure for designing composites with a unidimensional structure and prescribed averaged characteristics.

1. We equate the corresponding left sides of Eqs. (14)-(17) to the required values of the averaged characteristics c^0, a_{ij}^0, E_i^0, ν_{ij}^0, G_{ij}^0, A_{ij}^0. We obtain a system of algebraic equations in the variables X, Y, Z, x, y, z, t.
2. If the resulting system is not solvable or if the inequalities in (14)-(17) are not satisfied for its solution, then a material with the prescribed characteristics generally cannot be made within the given class of composites.
3. If the system is solvable and its solution satisfies the inequalities in (14)-(17), then a material with the required characteristics can be made within the class of composites with a unidimensional structure. To design such a material, it is sufficient to use the above-described methods [see Eqs. (6)-(13) and the accompanying text] to construct functions $(c(t), a(t), E(t), A(t)) \in W^4$ such that $\langle c \rangle = X$, $\langle 1/a \rangle = Y$, $\langle a \rangle =$ Z, $\langle E \rangle = x$, $\langle 1/E \rangle = y$, $\langle A \rangle = z$, $\langle EA \rangle = t$. The functions $c(t)$, $a(t)$, $E(t)$, $A(t)$, where $t \in [0, 1]$, give the distribution of the materials on the period of the composite ($t = x_1/\varepsilon$). A composite with a unidimensional periodic structure (with the period $\varepsilon \ll 1$), having local characteristics $c(x_1/\varepsilon)$, $a(x_1/\varepsilon)$, $E(x_1/\varepsilon)$, $A(x_1/\varepsilon)$, will by virtue of (1)-(4) and Propositions 1 and 2 have the required averaged characteristics c^0, a_{ij}^0, E_i^0, ν_{ij}^0, G_{ij}^0, A_{ij}^0.

Note 3. With the construction of the functions $c(t)$, $a(t)$, $E(t)$, $A(t)$ by the methods incorporated into (6)-(13), we obtain a design of a composite made of three materials. It can be seen from (10) and (13) that the quantities $\mu_1 = H/(1 + \delta_0)$, $\mu_2 = 1 - H/(1 + \delta_0)$, $\mu_3 = \delta_0/(1 + \delta_0)$ are equal to the volume contents of the materials which make up the composite.

Note 4. Equations (6)-(13) give one of the possible methods of obtaining a solution (obviously not the only method) of the problem of developing a composite with assigned averaged characteristics. Another method of solving this problem, following from "b" of Proposition 2, is looking for the solution in the class of composites formed of a finite number of materials (no less than three).

An example is the existence of a composite having a negative coefficient of linear expansion and made of components with positive coefficients of linear expansion (synthesis problem).

We examined composites made of components having coefficients of linear expansion $A(x_1/\varepsilon) > 0$ (i.e., components which expand with heating). Thanks to control over its local characteristics, the composite as a whole may be given properties which differ from the properties of its components. In particular, we will show that it is possible to obtain a negative A_{11} (i.e., to obtain a composite which will contract in the direction of the Ox_1 axis during heating). In accordance with the first equation of (17), the possible values of the averaged coefficient of linear expansion A_{11} are found from the equality

$$A_{11} = \frac{1+\nu}{1-\nu} z - \frac{2\nu}{1+\nu} \frac{t}{x} \quad (x > 0,\ z > 0,\ t > 0,\ y > 1/x). \tag{18}$$

TABLE 1

Composite	i	$E_i \cdot 10^{10}$, Pa	$A_i \cdot 10^{-6}$, K^{-1}	H_i	$A_{11} \cdot 10^{-6}$, K^{-1}
Iridium	1	52.8	6.5	0.1	—3.79
Invar	2	13.5	0.2	0.9	
Teflon	1	9,8	220	0.1	—49.0
Getinax	2	1,2	20	0.9	
Iridium	1	52.8	6.5	0.05	
Tungsten	2	39	4.5	0.05	—2.79
Invar	3	13.5	0.2	0.9	

It can be seen from (18) that the set of possible values of A_{11} is $(-\infty, +\infty)$. Thus, a composite having a negative coefficient A_{11} can exist.

Let us examine the problem of synthesizing a composite with a negative coefficient A_{11}. One method of solving this problem [on the basis of Eqs. (6)-(13)] was presented above; we will illustrate another method, mentioned in Note 4. In accordance with Proposition 2, all of the possible values of A_{11} can be obtained in the class of laminated composites made of no more than three materials. Let E_i, A_i, and H_i be the Young's modulus, the coefficient of linear expansion, and the volume content of the i-th material (i = 1, 2, 3). Then all possible values of A_{11} are given by Eq. (18), in which we should put

$$x=\sum_{i=1}^{3} E_iH_i, \quad y=\sum_{i=1}^{3} H_i/E_i,$$

$$z=\sum_{i=1}^{3} A_iH_i, \quad t=\sum_{i=1}^{3} E_iA_iH_i,$$

$$0\leqslant H_i\leqslant 1, \quad \sum_{i=1}^{3} H_i=1, \quad E_i, A_i>0.$$

The synthesis problem reduces in this case to a finite-dimensional problem. After its discretization, we arrive at a finite sorting problem to search for the sets of values of E_i, A_i, H_i (i = 1, 2, 3) giving the required values of A_{11}. The sorting was done on a BÉSM-6 computer. Working with the computer-generated solutions of the problem of synthesis of a composite with a negative coefficient A_{11} (the computer output on the order of 100 designs), we selected composites of actual materials having a negative coefficients A_{11}. These composites are shown in Table 1 (the composite is represented as a two-layer composite when the characteristics of two layer coincide).

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